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DETERMINATION OF NEAR WAKE SHEAR LAYER BY SHOCK EXPANSION METHOD FOR AXISYMMETRIC SLENDER BODY

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Abstract—This paper shows how the shock expansion theory can be applied to determine the shear layer, which is part of the near wake of an axisymmetric slender body flying at hypersonic speed. Previous investigators have shown that this flow region can be described by neglecting transport properties and thus applying the method of characteristics. This paper shows that the same flow pattern can be computed, to a reasonable degree of accuracy, by further neglecting one family of characteristics. Hence the overall structure of the shear layer is due to an expansion around the corner followed by a recompression due to axisymmetric forces, where the entropy is constant along each streamline. Furthermore, the characteristics coalesce after the neck and form a trailing shock. Numerical computations using this method are straightforward and fast.

1. INTRODUCTION

THE WAKE behind an axisymmetric slender body flying at hypersonic speed can be conveniently described by dividing the flow pattern into several regions (Fig. 1). At the sharp shoulder of the vehicle the boundary layer separates and contains a recirculation region where the velocity of the fluid with respect to the body is small. That region closes at a rear stagnation point; further downstream lies the neck and the viscous far wake. This paper shows that the shock expansion theory yields a straightforward and reasonably accurate computation of the shear layer for slender body.

In the early investigations of the shear layer it was assumed that the governing phenomenon was the viscous mixing of the shear layer with the outer and recirculation regions at nearly constant pressure. That basic idea was first suggested by Chapman [1] and was applied to this problem by Denison and Baum [2]. However subsequent data obtained by Pallone [3], Muntz [4] and Murman [5] showed an appreciable transverse pressure gradient and a shorter cavity than predicted by the mixing theory.

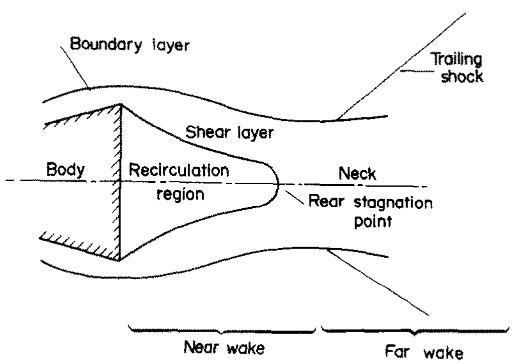

FIG. 1. Flow regions in the wake.

In another method the influence of the pressure field was taken into account by considering the interaction between viscous and inviscid flow at the outer edge of the shear layer by use of an integral method [6, 7]. The solution goes through a saddle point, which determines the location of the rear stagnation point and insures the uniqueness of the solution. The mathematical analysis is fairly involved, and the results are given in terms of integral quantities difficult to compare with some of the experimental data.

In another approach the dominant phenomenon is assumed to be rapid expansion of the boundary layer around the sharp shoulder of the vehicle. This was first suggested by Weinbaum [8] and the latest report on this subject was published by Weiss [9]. The expansion is too rapid to permit the transfer of vorticity between streamlines and thus the supersonic part of the shear layer can be determined by the method of characteristics. This yields the numerical values of the physical quantities of interest which can, at least in principle, be compared with experimental data. A great deal of effort is spent in computing the "lip shock" observed by Hama [10] and other observers at Mach numbers 2·5–5. Photographs taken by Larson [11] and Murman [5] at Mach numbers above 12 fail to indicate such a lip shock and its effect on the hypersonic slender flow pattern appears to be small.

These features are applicable when the Mach number at the edge of the boundary layer at separation is hypersonic. This happens in the case of slender body wakes. For blunt bodies the fluid is slowed down more drastically, and the Mach number at the edge of the boundary layer is in the low supersonic regime. Hence the inviscid approach is not applicable.

In the present calculation the overall structure of the shear layer is governed by (1) the inviscid rotational expansion of the boundary layer without secondary lip shock (2) the compression due to axisymmetric forces. A rotational shock expansion method is used.

The general aspect of the ensuing flow pattern is in good agreement with observation and more complicated calculations. The computed static temperature and pressure are within the uncertainties of the experimental data. Furthermore, it is found that the characteristics of the one family which is used coalesce around the neck region and thus determine the inner part of the trailing shock. The existence of the latter is confirmed by numerous data.

A flow model which has similarities with the present one has been the object of a detailed theoretical investigation, particularly near the shoulder, by Adamson [12].

The present method leads to straightforward fast numerical computations. This is particularly helpful when, for example, it is desired to obtain the flow field for a large number of different values of the governing parameters, or to include the effect of unsteady chemistry.

2. PRINCIPLE OF THE PRESENT METHOD

The two $C_\pm$ families of characteristics are specified by

—the angle between the characteristics and the reference axis, taken as the axis of symmetry for axisymmetric geometry

$$\theta \pm \mu$$

where θ is the angle between the streamline and the reference axis, and μ the local Mach angle.

—the analytic expression valid along each family is respectively

$$d\alpha_\pm = 0$$

where $d\alpha_\pm$ are defined by:

$$d\alpha_\pm \equiv \frac{dp}{\rho q^2 \tan\mu} \pm d\theta + \varepsilon \frac{\sin\theta \sin\mu}{\sin(\theta \pm \mu)} \frac{dy}{y}$$

p, ρ, q, y are respectively pressure, density, velocity vector and transverse coordinate. $\varepsilon = 0$ for two dimensional geometry and $\varepsilon = +1$ for axisymmetric geometry.

In the present approach the C_- family is ignored so that the flow field is assumed to be a one family region. This assumption is exact in the two dimensional Prandtl Meyer expansion.

In the method of characteristics computation the flow field is supposed to be known up to the data line LZN (Fig. 2). The flow field at P is determined by applying $d\alpha_\pm = 0$ along NP and LP respectively and by writing that the entropy at P is the same as at Z.

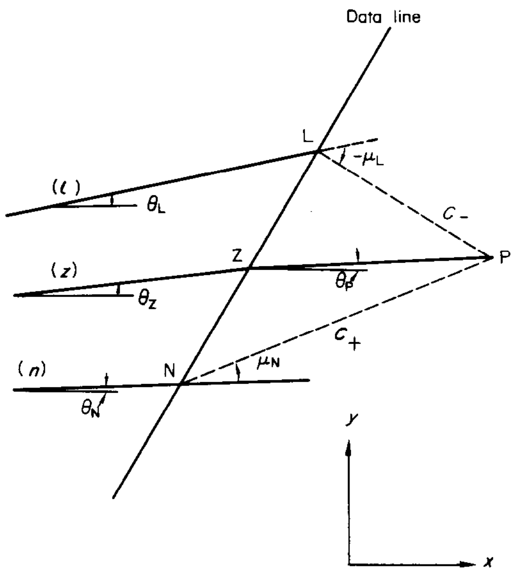

FIG. 2. Method of characteristics computation.

In the present computational procedure $d\alpha_- = 0$ is applied along the streamline ZP instead of the characteristic LP. This means that as far as the C_- influence at P is concerned, the flow properties at L are assumed to be the same as at Z.

The computation starts at the separation point on the sonic streamline. The generation of the C_+ characteristics is obtained by assuming that the sonic streamline undergoes a series of infinitesimal turns oriented toward the cavity. Then the flow is computed successively along each characteristic from the sonic up to the outer streamline. (Fig. 3)

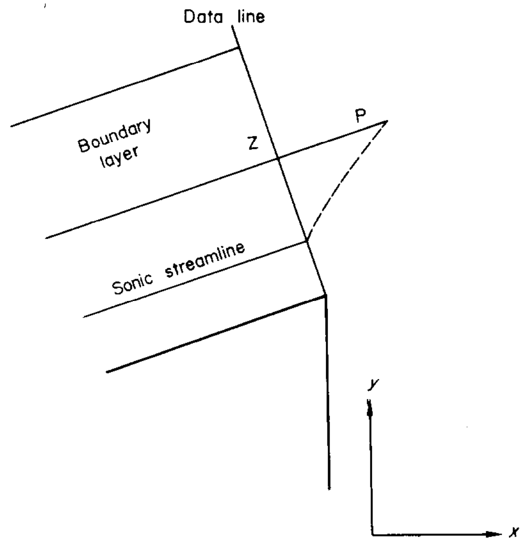

FIG. 3. Application of simplified method of characteristics to boundary layer expansion.

3. UNIQUENESS

When the boundary layer profile has been specified, the flow field is entirely determined by the turning angle of the sonic streamline at separation.

For the two dimensional computation this angle is determined by a prescribed aft pressure.

For the axisymmetric case, the determination of this angle should be self-contained. When the angle is large enough, the inner characteristics begin by turning toward the axis, reaching a point of minimum ordinate. Just downstream of this point the characteristics move away from the axis and coalesce along the characteristic which was innermost upstream. In that region the elementary pressure jumps across each characteristic accumulate and generate the trailing shock. By applying the Rankine–Hugoniot equations along the innermost characteristic, it is found that the generation of the trailing shock begins at a point slightly aft of the point of minimum ordinate. The turning angle of the sonic streamline at separation is selected in order to render the flow after the trailing shock parallel to the axis.

4. RESULTS

The method described above has been applied to compute flow fields with two dimensional and axisymmetric geometries; and the results are compared with computational data obtained by more complicated methods.

4.1 *Two dimensional results*

In the two dimensional flow field the dominant phenomenon is a monotonic expansion where the entropy remains constant along each streamline; the pressure can only decrease since the flow model does not contain any compression mechanism. A typical flow pattern is depicted in Fig. 4. The sonic streamline is a straight line

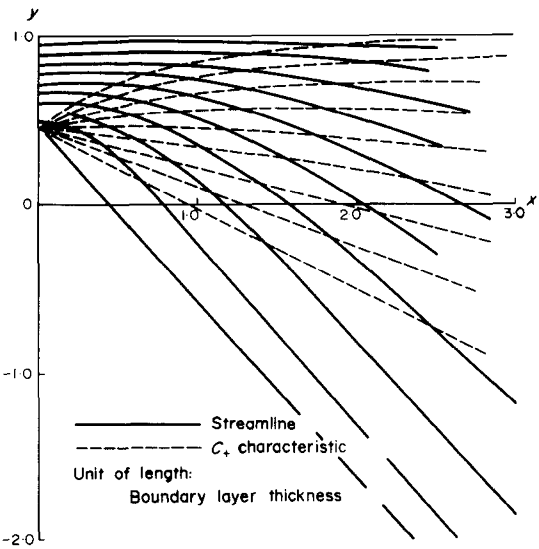

FIG. 4. Typical flow pattern for two-dimensional geometry.

after the expansion. The C_+ characteristics are expansion waves, their concavity is oriented toward the body. Figure 5 shows the variations of flow quantities along streamlines, Fig. 5a depicts the variations of the turning angle $(-\theta)$ which tends monotonically towards the maximum Prandtl Meyer turning angle that the considered streamline can turn. Figure 5b depicts the variations of pressure which tends monotonically toward zero.

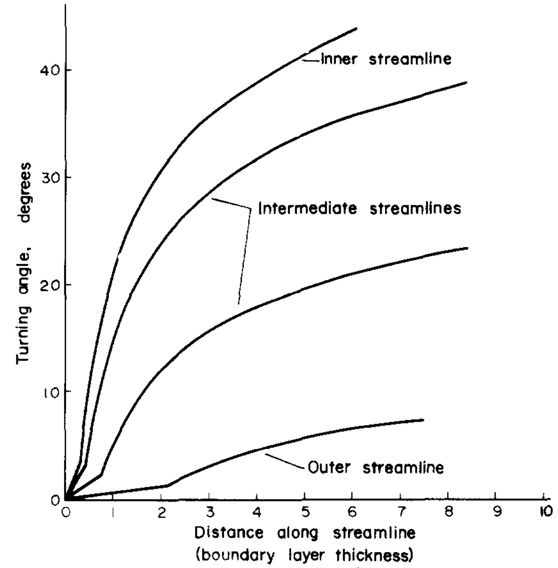

FIG. 5a. Variations of turning angle along streamlines. Abscissa: Arc length since separation point; Unit: Boundary layer thickness.

The present method has been applied with the same input parameters as the computation using the two families of characteristics [8].

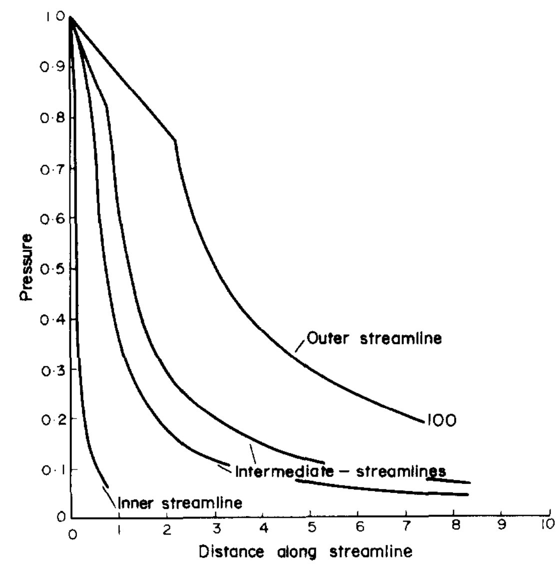

FIG. 5b. Variations of pressure along streamline. Abscissa: Same as for Fig. 5a. Unit for ordinate: Pressure in boundary layer.

Figure 6 shows the variations of the flow turning angle along the C_+ characteristics obtained by each of the methods. The Mach number at separation, taken as the abscissa specifies a particular streamline. The shock expansion method yields a bigger turning angle than the two characteristics system. Thus the effect of neglecting the C_- characteristics is to turn the flow more rapidly.

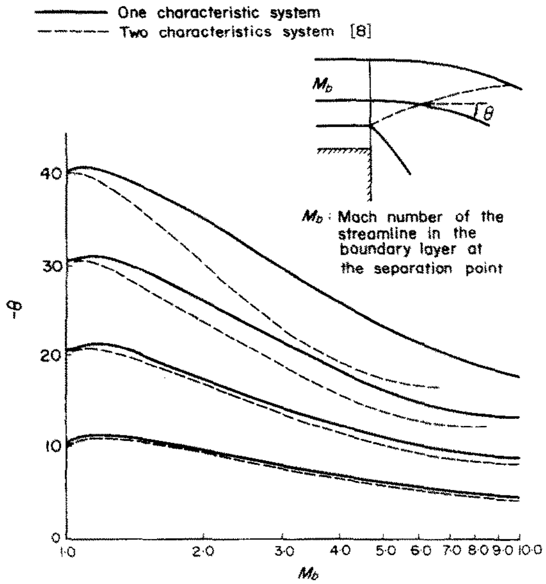

FIG. 6. Variations of the turning angle of the streamlines along the C_+ characteristics.

4.2 *Axisymmetric results*

The axisymmetric shear layer is determined by an expansion around the shoulder of the vehicle, followed by a compression due to axisymmetric forces. These two phenomena occur at constant entropy and total enthalpy along each streamline.

A typical flow pattern is depicted in Figs. 7 and 8. At first sight the pictures given by the computed flow pattern agree qualitatively with experimental observation. The boundary layer expands into a thick shear layer which almost closes itself at the neck. The outer streamlines and the first few characteristics are not greatly affected by axisymmetric forces, as might be expected. However, subsequent characteristics straighten out and then reverse the initial orientation of their concavity.

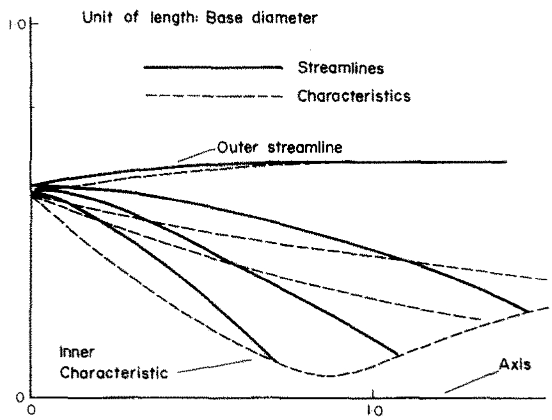

FIG. 7. Typical flow pattern for axisymmetric geometry. Unit: Base diameter.

Furthermore, it is found that the point of

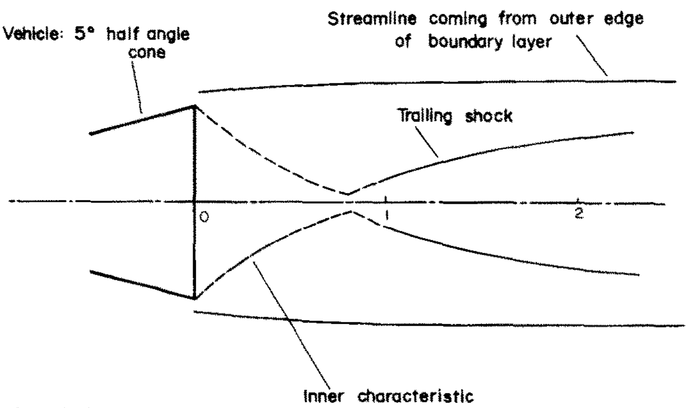

FIG. 8. Geometrical configuration of the trailing shock as determined by present method.

minimum abscissa for the innermost characteristic, which corresponds to the neck region, is located at approximately 0·8 base diameter. This result has been confirmed by numerous computations and experimental data.

The variations of flow parameters along streamlines is shown in Fig. 9. Near the separation the variations of the flow parameters are not significantly affected by the axisymmetric forces and it remains so downstream for the flow near the outer streamline. The flow near the inner characteristic undergoes a strong compression and the pressure rises sharply.

Figure 10 indicates the variations of flow parameters along the trailing shock. The combination of increasing Mach number and decreasing shock angle generates a shock which is weak near the axis and gains strength when propagating outward. The pressure ratio starts at unity and tends asymptotically to a finite value of about 3·2. Published computational data [9] obtained by the method of characteristics are concerned with the distribution of flow parameters along the axis, and since the present method excludes the region near the axis, a direct comparison is difficult. However, it is reasonable to assume that the pressure increase along the axis reported in [9] is due to the trailing shock. Therefore the ratio between the peak pressure which occurs at about two base diameters aft of the body, to the pressure at, say 0·7 base diameter, is a measure of the strength of the trailing shock. [9] indicates that this pressure ratio is approximately 3·25 which agrees well with the present value of 3·2.

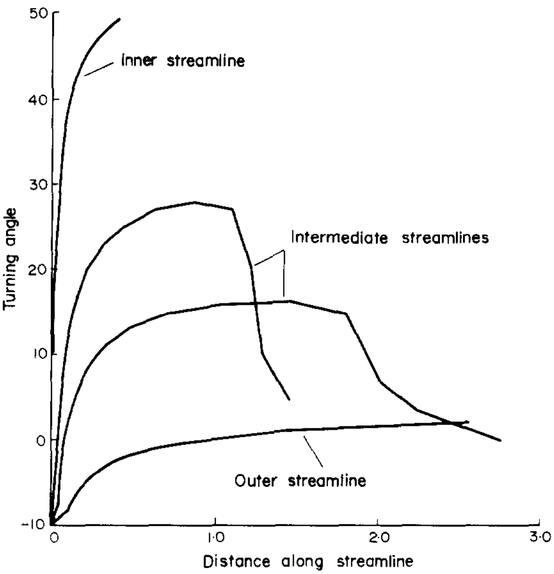

FIG. 9a. Variations of turning angle along streamlines. Abscissa: Arc length since separation profile, unit base diameter.

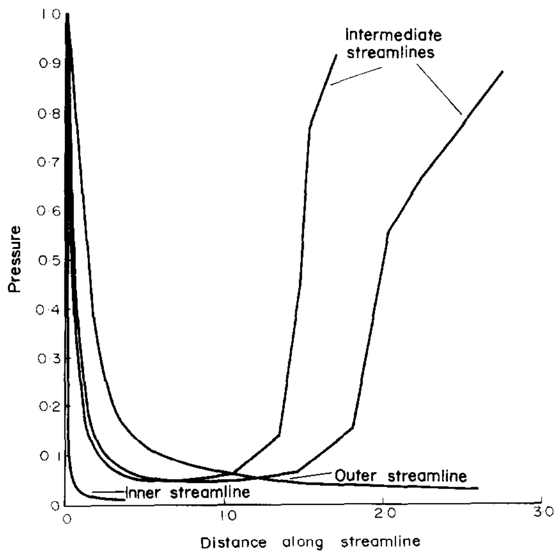

FIG. 9b. Pressure along streamlines. Same abscissa as Fig. 9a. Pressure unit: Pressure in the boundary layer.

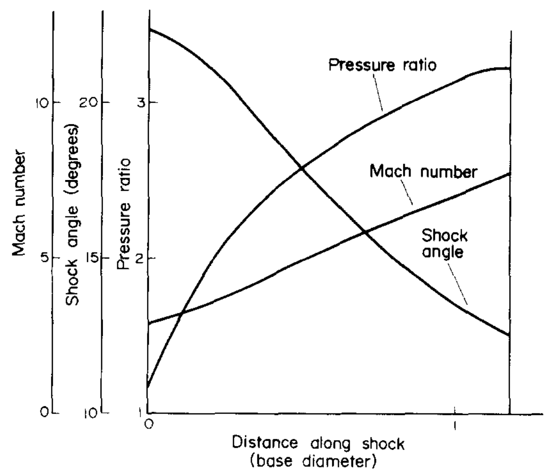

FIG. 10. Variations of flow parameters along trailing shock.

5. COMPARISON WITH EXPERIMENTAL DATA

The present method has been applied to determine flow fields corresponding to published experimental conditions. The static pressure is compared in Fig. 11, which is a reproduction of Fig. 2 of [3] where the results of computations have been added. The computation points fall into a region which might be considered as within the experimental uncertainties.

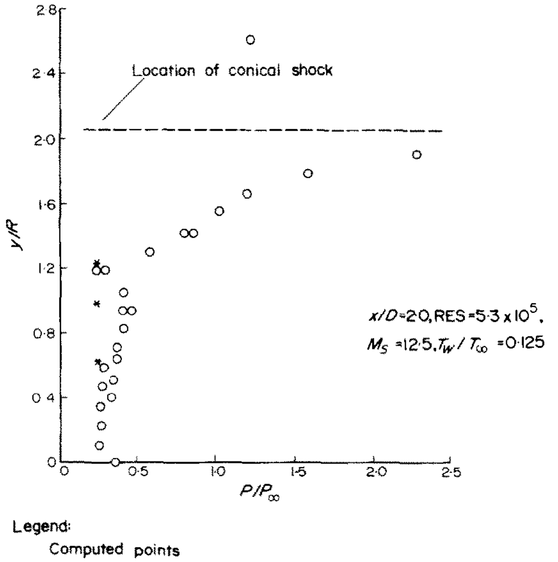

FIG. 11. Static pressure radial profile for a 5° half-angle cone (Fig. 2 of [3]).

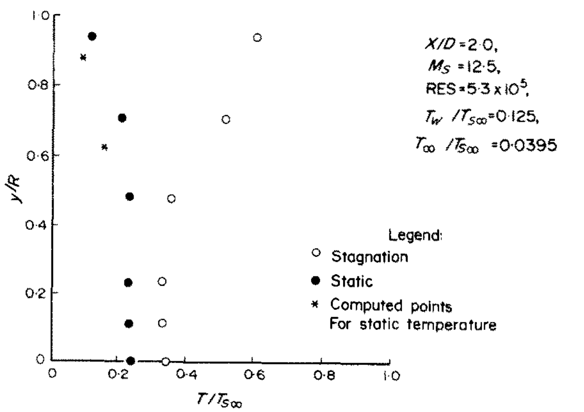

FIG. 12. Radial temperature profile for a 5° half-angle cone (Fig. 5 of [3]).

The temperature is compared in Fig. 12, which is a reproduction of Fig. 5 of [3]. One notices that this comparison is for $x/D = 2$ which shows that the validity of the method extends beyond the neck, provided the area considered is away from the axis and more specifically above the trailing shock.

Comparison between measured and computed static temperature and pressure has also been carried out for some values of the flow parameters reported in [4]. Figure 13 establishes

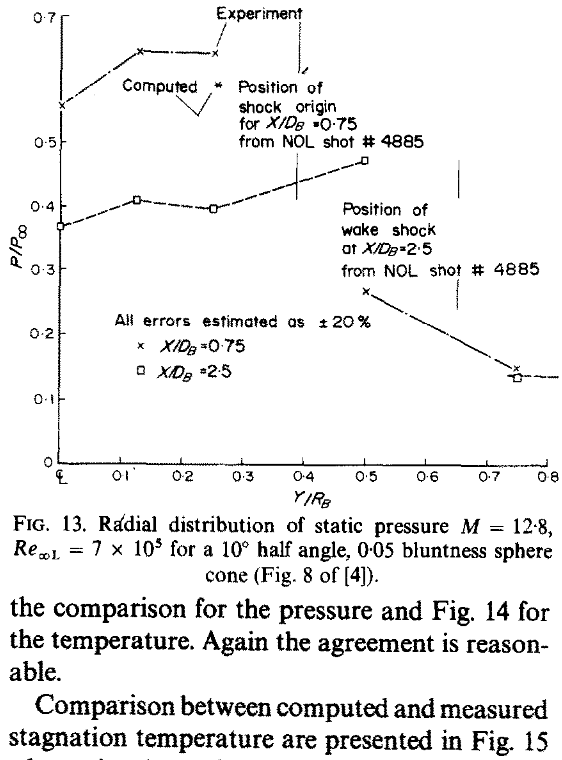

FIG. 13. Radial distribution of static pressure $M = 12{\cdot}8$, $Re_{\infty L} = 7 \times 10^5$ for a 10° half angle, 0·05 bluntness sphere cone (Fig. 8 of [4]).

the comparison for the pressure and Fig. 14 for the temperature. Again the agreement is reasonable.

Comparison between computed and measured stagnation temperature are presented in Fig. 15 where the determining flow parameters come from [3].

As can be anticipated, a significant difference appears between the computed and measured values of temperature near the axis. Typically at a point located at $(x/D) = 0{\cdot}8$ $(y/D) = 0{\cdot}02$ the computed temperature is 50 per cent higher than the measured one. This is due to the effect of the transport properties which are expected to be significant near the axis and are not taken into account in the present model.

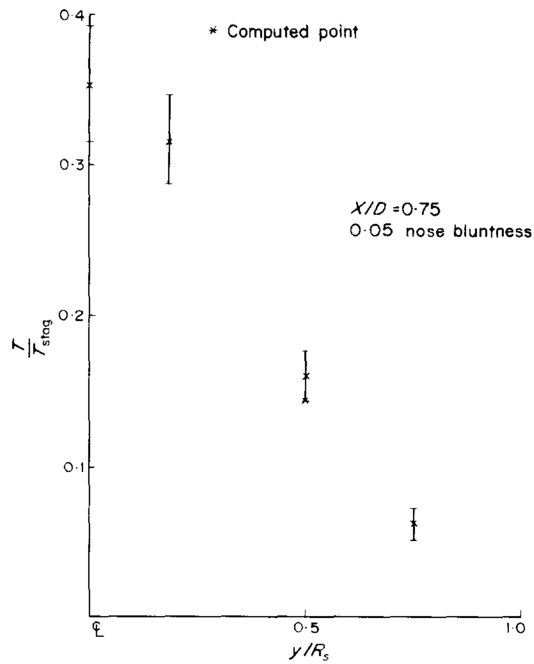

FIG. 14. Radial distribution of static temperature for 10° cone with 0·05 bluntness. $M_\infty = 12{\cdot}8$, $Re_{\infty D} = 2 \times 10^5$. (Fig. 16 of [4]).

The location of the trailing shock found by the present method agrees with experimental observation [5].

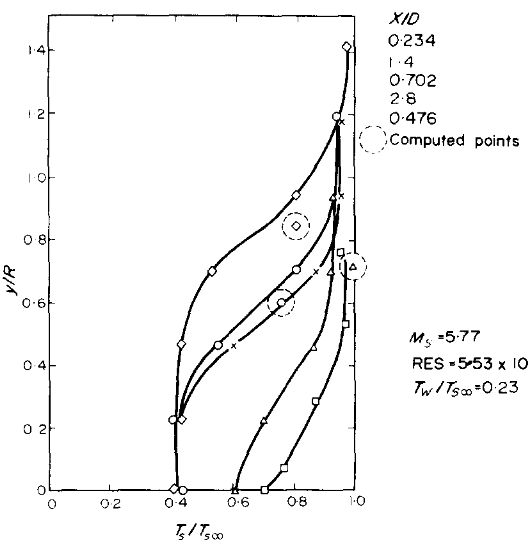

FIG. 15. Stagnation temperature radial profiles for a 5° half angle cone (Fig. 7 of [3]).

6. CONCLUSION

For the flow regime where the effect of transport properties can be neglected the rapid deflection of a boundary layer over a step is governed by a monotonic expansion where the entropy remains constant along each streamline. The flow field can be described by the shock expansion theory.

The axisymmetric shear layer is governed by the same kind of expansion combined with the compression due to axisymmetric forces. The application of the shock expansion theory gives results in agreement with more complete calculations and experimental observations. The numerical computations using the present method are straightforward and more rapid than by utilizing the method of characteristics.

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DÉTERMINATION DE LA COUCHE DE CISAILLEMENT PROCHE DU SILLAGE PAR LA MÉTHODE DE DÉTENTE POUR UN CORPS EFFILÉ À SYMÉTRIE AXIALE

Résumé—Cet article montre comment la théorie de l'expansion de choc peut être appliquée pour déterminer la couche de cisaillement qui est la région proche du sillage d'un corps effilé axisymétrique se déplaçant à vitesse hypersonique. Des chercheurs antérieurs ont montré que cette région peut être décrite en négligeant les propriétés de transport et en appliquant la méthode des caractéristiques. Cet article montre que la même configuration d'écoulement peut être calculée avec un degré raisonnable de précision en négligeant une famille de caractéristiques. Par suite la structure de la couche de cisaillement est due à une détente autour du coin suivie d'une recompression due aux forces axisymétriques l'entropie restant constante le long de chaque ligne de courant. Les caractéristiques se condensent pour former une onde de bord de fuite. Les calculs numériques utilisant cette méthode sont directs et rapides.

BESTIMMUNG DER SCHERSTRÖMUNG IM NACHLAUFGEBIET VON ACHSSYMMETRISCHEN SCHLANKEN KÖRPERN NACH DER STOSSWELLENMETHODE

Dieser Beitrag zeigt, wie die Theorie der Stosswellenausbreitung angewendet werden kann, um die Scherströmung zu bestimmen, die Teil der nahen Nachlaufströmung eines achsensymmetrischen, schlanken Körpers ist, der mit Überschallgeschwindigkeit fliegt. Frühere Untersuchungen haben gezeigt, dass diese Strömungszone beschrieben werden kann, indem Transportgrössen vernachlässigt werden und somit die Methode der Charakteristiken angewendet werden kann. Dieser Beitrag zeigt nun, dass dasselbe Strömungsprofil mit einer genügend grossen Genauigkeit auch errechnet werden kann, wenn eine Gruppe der Charakteristika vernachlässigt wird. Somit ist die Gesamtstruktur der Scherströmung bedingt durch eine Expansion an der Kante gefolgt von einer erneuten Verdichtung durch achsensymmetrische Kräfte, wobei die Entropie entlang der Stromlinien konstant ist. Des weiteren vereinigen sich die Charakteristika hinter dem Hals und bilden einen nachlaufenden Stoss. Die numerischen Berechnungen mit Hilfe dieser Methode sind direkt und schnell.

ОПРЕДЕЛЕНИЕ СДВИГОВОГО СЛОЯ БЛИЖНЕГО СЛЕДА МЕТОДОМ УДАРНОГО РАСШИРЕНИЯ ДЛЯ ОСЕСИММЕТРИЧНОГО ТОНКОГО ТЕЛА

Аннотация—В статье показано, как можно применить теорию ударного расширения для определения сдвигового слоя, который является частью ближнего следа осесим метричного тонкого тела, движущегося с гиперзвуковой скоростью. Предыдущие исследователи показали, что эта область течения может быть описана, если пренебречь свойствами переноса и, таким образом, применить метод характеристик. Показано также, что ту же самую модель течения можно рассчитать с достаточной степенью точности, пренебрегая одним семейством кривых. Поэтому общая структура сдвигового слоя определяется расширением возле угла, предшествующего рекомпрессии, которая является следствием воздействия осесимметричных сил и где энтропия постоянна вдоль каждой линии тока. Кроме того, характеристики смыкаются за сужением и образуют хвостовую ударную волну. Численные расчёты с помощью этого метода проводятся непосредственно и очень быстро.